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Estimates and Recommendations for Coincidence Geometry

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Estimates and recommendations for coincidence geometry

W. Younes and J. J. Ressler
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I. ROUGH ESTIMATE OF COINCIDENCE SUMMING CORRECTIONS

A. Introduction

When two truly coincident gamma-rays deposit their energy within the same detector, a composite pulse which is indistinguishable from one due to a single event may be recorded by that detector. This summing effect is known to become more important as the distance from source to detector is decreased [1]. In this short report, we give a rough estimate for the size of this effect as a function of source-to-detector distance. The formalism used in this report is taken mainly from [2], and similar results can also be found, e.g., in [1, 3, 4]. In general, the size of the effect will depend on the exact level scheme of the nucleus studied, but for the sake of extracting numerical values, we will assume a particular level scheme in this report.

B. Assumed measurement conditions

1. Simplified level scheme

For demonstration purposes, we assume the simple level scheme shown in Fig. 3 of [2], and reproduced as Fig. 1 of this report for easy reference. The energy of each gamma-ray is denoted by E_i, and its emission probability is given by p_i. The side-feeding probabilities are β_i, with $\beta_1 + \beta_2 + \beta_3 = 1$. Thus, if we denote by b_i the fraction of decays from the level from which γ_i originates and which produces the γ_i photons (i.e., the branching ratios), we have the following relations[5]

$$\begin{aligned} p_1 &= \beta_2 b_1 \\ p_2 &= (\beta_2 b_1 + \beta_1)\, b_2 \\ p_3 &= \beta_2 b_3 \end{aligned}$$

In the numerical applications below, we will assume $\beta_1 = 0$, and $b_1 = 2/3$, $b_2 = 1$, $b_3 = 1/3$, and therefore

$$\begin{aligned} p_1 &= \frac{2}{3}\beta_2 \\ p_2 &= \frac{2}{3}\beta_2 \\ p_3 &= \frac{1}{3}\beta_2 \end{aligned}$$

We do not need to supply a numerical value for β_2, because the emission probabilities p_i will always appear as ratios in the calculations below.

2. Simplified detector

In order to simplify the efficiency calculations, we approximate the Ge crystal by a flat disk (rather than a thick cylinder) of diameter a. For numerical applications, we will use $a = 11.4\,\mathrm{cm}$ (the diameter for the BE6530 model). A schematic representation of this detector, located at a distance d from a source, is shown in Fig. 2. The solid angle subtended by the disk is given by

$$\Omega = \int\!\!\int \frac{\hat{n} \cdot d\vec{a}}{r^2}$$

where $\hat{n}$ is a unit vector from the origin to a point on the surface, $d\vec{a}$ is a vector whose magnitude is a differential element of area and whose direction is perpendicular to the surface. Finally, r is the distance from the origin to a point

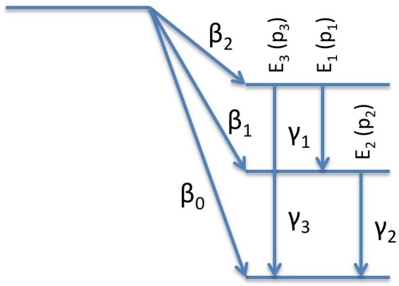

Figure 1: Simplified level scheme used in this report.

on the surface. In the special case of the disk in Fig. 2 this reduces to the double integral in cylindrical coordinates

$$\Omega = \int_0^{2\pi} d\theta \int_0^{a/2} \rho\, d\rho \, \frac{\cos\phi}{d^2+\rho^2}$$

which can be evaluated analytically

$$\begin{aligned} \Omega &= 2\pi d \int_0^{a/2} \frac{\rho\, d\rho}{(d^2+\rho^2)^{3/2}} \\ &= 2\pi \left(1 - \frac{1}{\sqrt{1+b^2}}\right) \end{aligned} \tag{1}$$

and where we have introduced the dimensionless parameter

$$b \equiv \frac{a}{2d}$$

Given a diameter a and distance d of the detector, we can therefore readily calculate its geometric efficiency

$$\varepsilon_{\text{geo}} \equiv \frac{\Omega}{4\pi} = \frac{1}{2}\left(1 - \frac{1}{\sqrt{1+b^2}}\right) \tag{2}$$

The intrinsic efficiency ε_{int} will normally depend on the gamma-ray energies, but since these will vary depending on the nucleus under consideration, we will assume that it is independent of gamma-ray energy, and in numerical applications below we will take $\varepsilon_{\text{int}} = 0.6$. We will also make a distinction between the peak efficiency (ε_i with $i = 1, 2, 3$) for detecting a gamma-ray at a precise energy, and the total efficiency (ε_i^T with $i = 1, 2, 3$) for detecting a gamma-ray at any energy in the spectrum (e.g., if it does not deposit its full energy in the spectrometer). In this case, we have the relations

$$\varepsilon_i = \varepsilon_{\text{geo}} \times \varepsilon_{\text{int}}$$

and

$$\varepsilon_i^T = \frac{\varepsilon_i}{P/T}$$

where P/T is the peak-to-total ratio which, for numerical applications, we will take equal to 0.7.

C. Calculation of correction and uncertainty

1. *Formulas for coincidence-sum correction factors*

We now recapitulate the results in [2]. For γ_1, if there were no cascade, and given an activity A of the nucleus feeding the levels in Fig. 1, we would expect a count rate

$$N_1 = A p_1 \varepsilon_1$$

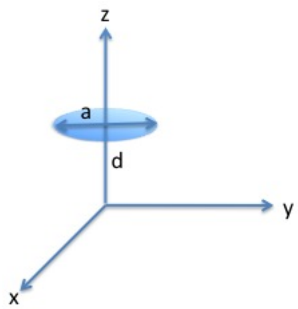

Figure 2: Disk-shaped detector of diameter a, at a distance d from the source.

for the photopeak of this gamma-ray. However, every time γ_1 is emitted, γ_2 will also be emitted. Thus, there is a probability that the energy of γ_2 will also be recorded (in part or in full) at the same time as that of γ_1. For simplicity, we ignore any angular correlation effects. Then, the number of times we record simultaneously the sum of E_1 and either all or part of E_2 is given by $N_1\varepsilon_2^T$. Whenever this happens, we lose counts in the γ_1 photopeak, and observe a smaller number of counts

$$N_1' = N_1 - N_1\varepsilon_2^T$$

This loss must be accounted for by a correction factor

$$\boxed{C_1 \equiv \frac{N_1}{N_1'} = \frac{1}{1-\varepsilon_2^T}}$$

Next, for γ_2, if there were no cascade, we would expect a count rate

$$N_2 = Ap_2\varepsilon_2$$

Because of the $\gamma_1 - \gamma_2$ coincidence, we expect events were the sum of E_2 and either all or part of E_1 is recorded, taking counts away from the γ_2 photopeak. Again, ignoring angular-correlation effects, these sum-coincidence events will occur at a rate $\left(Ap_1\varepsilon_1^T\right)\varepsilon_2$, and therefore we will observe a rate in the γ_2 photopeak of

$$\begin{aligned} N_2' &= N_2 - \left(Ap_1\varepsilon_1^T\right)\varepsilon_2 \\ &= N_2\left(1 - \frac{p_1}{p_2}\varepsilon_1^T\right) \end{aligned}$$

The loss in photopeak counts must be accounted for by the correction factor

$$\boxed{C_2 \equiv \frac{1}{1-\frac{p_1}{p_2}\varepsilon_1^T}}$$

Finally, for γ_3, the situation is slightly different. In this case, it is possible to detect the full-energy sum $E_1 + E_2$ from a $\gamma_1 - \gamma_2$ coincidence which would be erroneously attributed to a γ_3 decay, thereby leading to a peak gain in the

γ_3 photopeak. The observed counts, taking this effect into account, would be[6]

$$\begin{aligned} N_3' &= N_3 + A p_1 \varepsilon_1 \varepsilon_2 \\ &= N_3 \left(1 + \frac{p_1 \varepsilon_1 \varepsilon_2}{p_3 \varepsilon_3}\right) \end{aligned}$$

giving a peak-gain correction factor

$$\boxed{C_3 \equiv \frac{1}{1 + \frac{p_1 \varepsilon_1 \varepsilon_2}{p_3 \varepsilon_3}}}$$

2. Uncertainties for coincidence-sum correction factors

As stated in section I B 2, we will make the simplifying assumption that the peak and total efficiencies are independent of gamma-ray energy, i.e., $\varepsilon_1 = \varepsilon_2 = \varepsilon_3 \equiv \varepsilon$ and $\varepsilon_1^T = \varepsilon_2^T = \varepsilon_3^T \equiv \varepsilon^T$. In that case, the correction factors derived in section I C 1 reduce to

$$\begin{aligned} C_1 &= \frac{1}{1 - \varepsilon^T} \\ C_2 &= \frac{1}{1 - \frac{p_1}{p_2}\varepsilon^T} \\ C_3 &= \frac{1}{1 + \frac{p_1}{p_3}\varepsilon} \end{aligned}$$

The variance of C_1 is given by

$$\sigma_{C_1}^2 = \left(\frac{\partial C_1}{\partial \varepsilon^T}\right)^2 \sigma_{\varepsilon^T}^2$$

and, after straightforward calculations we get

$$\boxed{\frac{\sigma_{C_1}}{C_1} = \underbrace{\left(\frac{\varepsilon^T}{1 - \varepsilon^T}\right)}_{\rho_1} \frac{\sigma_{\varepsilon^T}}{\varepsilon^T}}$$

This very convenient form relates the relative uncertainty in C_1 to the relative uncertainty in the total efficiency ε^T through a scale factor ρ_1. We can obtain similar expressions for the other correction factors (making the reasonable assumption that efficiencies and branching ratios are uncorrelated). To wit,

$$\boxed{\frac{\sigma_{C_2}}{C_2} = \underbrace{\left(\frac{\frac{p_1}{p_2}\varepsilon^T}{1 - \frac{p_1}{p_2}\varepsilon^T}\right)}_{\rho_2} \sqrt{\left(\frac{\sigma_{p_1}}{p_1}\right)^2 + \left(\frac{\sigma_{p_2}}{p_2}\right)^2 + \left(\frac{\sigma_{\varepsilon^T}}{\varepsilon^T}\right)^2}}$$

and

$$\boxed{\frac{\sigma_{C_3}}{C_3} = \underbrace{\left(\frac{\frac{p_1}{p_3}\varepsilon}{1 + \frac{p_1}{p_3}\varepsilon}\right)}_{\rho_3} \sqrt{\left(\frac{\sigma_{p_1}}{p_1}\right)^2 + \left(\frac{\sigma_{p_2}}{p_2}\right)^2 + \left(\frac{\sigma_{\varepsilon}}{\varepsilon}\right)^2}}$$

Ideally, the experiment should be designed so that $\rho_i \ll 1$, and that therefore the coincidence-sum corrections contribute very little to the overall uncertainty, compared to the efficiencies and branching ratios.

Parameter	Value
a	11.4 cm
p_1	$\frac{2}{3}\beta_2$
p_2	$\frac{2}{3}\beta_2$
p_3	$\frac{1}{3}\beta_2$
ε_{int}	0.6
P/T	0.7

Table I: Parameter values used in the numerical calculations.

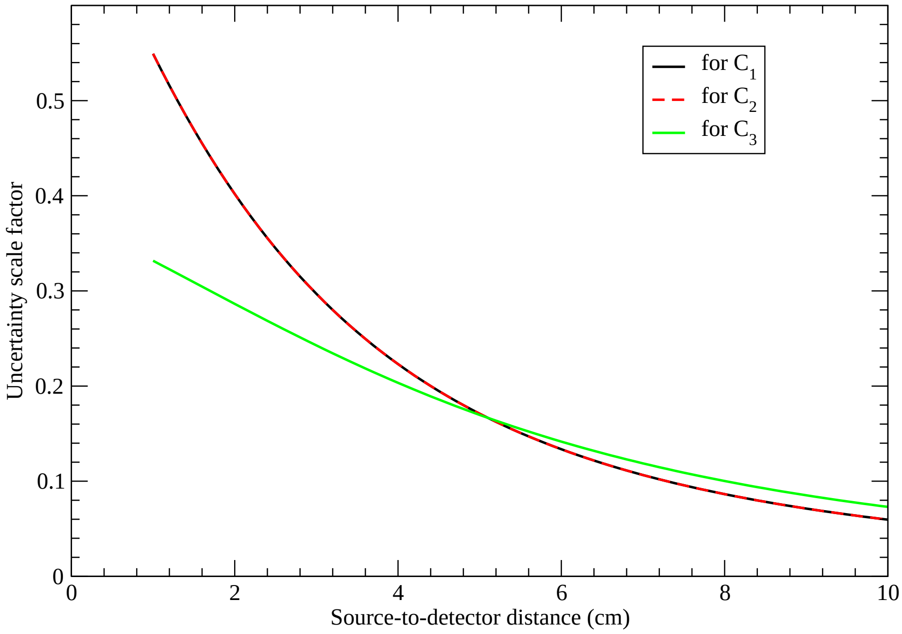

Figure 3: Uncertainty scale factors ρ_i plotted as a function of the distance d between source and detector. Note that, because $p_1 = p_2$ and all the efficiencies are the same in our example, the curves for C_1 and C_2 lie on top of each other.

3. Numerical application

Here we plot the uncertainty scale factors ρ_i derived in section I C 2, using the particular values of the parameters given in sections I B 1 and I B 2, and summarized in table I. These uncertainty scale factors are plotted in Fig. 3.

D. Recommendations

The results obtained in this report and summarized in Fig. 3 depend sensitively on the particular level scheme under consideration. We note in particular, that the factor ρ_3 tends to its maximum value of 1 for $b_1 \gg b_3$. In other words, if the γ_3 branch in Fig. 1 is very weak, then whenever we record an energy E_3 in the detector, it is far more likely to be caused by the $\gamma_1 - \gamma_2$ summed coincidence, with energy $E_1 + E_2 = E_3$, than by an actual γ_3 photon. In that case, the uncertainty contribution from the peak-gain correction factor C_3 would be as large as that of the other sources of uncertainty in the problem. In our particular example, we find that the correction uncertainties become small (i.e., $\rho_i \sim 10\%$) for a distance $d \gtrsim 10\,\text{cm}$. In practice, the distance d should be selected with the specific gamma-rays of interest and associated decay schemes in mind, with the goal of maximizing the geometric efficiency while keeping the coincidence-sum corrections at an acceptably small level.

II. SCATTER

A. Introduction

The detector will be sensitive the gamma-rays emitted by the source, as well as any photons scattered into the detector volume from surrounding materials. The more material surrounding the detection system, the large the scatter contribution to the energy spectrum. The energy of the scattered photon (E') depends on the energy of the originating gamma-ray (E) as well as the scattering angle (θ):

$$E' = \frac{E}{1 + \frac{(1-cos\theta)E}{m_e c^2}}$$

where m_e is the rest mass of the electron (0.511 MeV). Higher angle scatter, $>$90°, can significantly increase the count contribution to the lower energies of the gamma-ray spectrum. Very high scatter, at $\sim$180°, produces a broad backscatter peak that may confuse peak fitting, activity estimates, and nuclide identification efforts.

B. Recommendation

With active samples, significant scatter from shielding materials near the detector volume can hinder spectroscopic analyses. Therefore, the detectors should not have any additional materials nearby.

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[1] G. J. McCallum and G. E. Coote, Nucl. Instr. Meth. 130, 189-197 (1975).
[2] K. Debertin and U. Schötzig, Nucl. Instr. Meth. 158, 471-477 (1979).
[3] R. J. Gehrke, R. G. Helmer, and R. C. Greenwood, Nucl. Instr. Meth. 147, 405-423 (1977).
[4] S. I. Vasil'ev, et al., Instr. Expt. Tech. 49, 34-40 (2005).
[5] We ignore conversion electrons in the present estimates, but a correction for this effect could also be included in the formalism (see, e.g., [1]).
[6] Note that we use the peak efficiencies ε_i instead of the total efficiencies ε_i^T here because we are only interested in gamma-rays that deposit their full energy.